

Interacting holographic dark energy model in non-flat universe

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Abstract

We employ the holographic model of interacting dark energy to obtain the equation of state for the holographic energy density in non-flat (closed) universe enclosed by the event horizon measured from the sphere of horizon named L .

arXiv:hep-th/0609069 v2 14 Oct 2006

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1 Introduction

The accelerated expansion that based on recent astrophysical data [1], our universe is experiencing is today's most important problem of cosmology. Missing energy density - with negative pressure - responsible for this expansion has been dubbed Dark Energy (DE). Wide range of scenarios have been proposed to explain this acceleration while most of them can not explain all the features of universe or they have so many parameters that makes them difficult to fit. The models which have been discussed widely in literature are those which consider vacuum energy (cosmological constant) [2] as DE, introduce fifth elements and dub it quintessence [3] or scenarios named phantom [4] with $w < -1$, where w is parameter of state.

An approach to the problem of DE arises from holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. It was shown by 'tHooft and Susskind [5] that effective local quantum field theories greatly overcount degrees of freedom because the entropy scales extensively for an effective quantum field theory in a box of size L with UV cut-off Λ . As pointed out by [6], attempting to solve this problem, Cohen *et al.* showed [7] that in quantum field theory, short distance cut-off Λ is related to long distance cut-off L due to the limit set by forming a black hole. In other words the total energy of the system with size L should not exceed the mass of the same size black hole i.e. $L^3 \rho_\Lambda \leq LM_p^2$ where ρ_Λ is the quantum zero-point energy density caused by UV cutoff Λ and M_P denotes Planck mass ($M_p^2 = 1/G$). The largest L is required to saturate this inequality. Then its holographic energy density is given by $\rho_\Lambda = 3c^2 M_p^2 / L^2$ in which c is free dimensionless parameter and coefficient 3 is for convenience.

As an application of holographic principle in cosmology, it was studied by [8] that consequence of excluding those degrees of freedom of the system which will never be observed by that effective field theory gives rise to IR cut-off L at the future event horizon. Thus in a universe dominated by DE, the future event horizon will tend to constant of the order H_0^{-1} , i.e. the present Hubble radius. The consequences of such a cut-off could be visible at the largest observable scales and particularly in the low CMB multipoles where we deal with discrete wave numbers. Considering the power spectrum in finite universe as a consequence of holographic constraint, with different boundary conditions, and fitting it with LSS, CMB and supernova data, a cosmic duality between dark energy equation of state and power spectrum is obtained that can describe the low l features extremely well.

Based on cosmological state of holographic principle, proposed by Fischler and Susskind [9], the Holographic model of Dark Energy (HDE) has been proposed and studied widely in the literature [10, 11]. In [12] using the type Ia supernova data, the model of HDE is constrained once when c is unity and another time when c is taken as free parameter. It is concluded that the HDE is consistent with recent observations, but future observations are needed to constrain this model more precisely. In another paper [13], the anthropic principle for HDE is discussed. It is found that, provided that the amplitude of fluctuation are variable the anthropic consideration favors the HDE over the cosmological constant.

In HDE, in order to determine the proper and well-behaved system's IR cut-off, there are some difficulties that must be studied carefully to get results adapted with experiments that claim our universe has accelerated expansion. For instance, in the model proposed

by [10], it is discussed that considering particle horizon, R_p ,

$$R_p = a \int_0^t \frac{dt}{a} = a \int_0^a \frac{da}{Ha^2} \quad (1)$$

as the IR cut-off, the HDE density reads to be

$$\rho_\Lambda \propto a^{-2(1+\frac{1}{c})}, \quad (2)$$

that implies $w > -1/3$ which does not lead to accelerated universe. Also it is shown in [14] that for the case of closed universe, it violates the holographic bound.

The problem of taking apparent horizon (Hubble horizon) - the outermost surface defined by the null rays which instantaneously are not expanding, $R_A = 1/H$ - as the IR cut-off in the flat universe, was discussed by Hsu [15]. According to Hsu's argument, employing Friedman equation $\rho = 3M_P^2 H^2$ where ρ is the total energy density and taking $L = H^{-1}$ we will find $\rho_m = 3(1 - c^2)M_P^2 H^2$. Thus either ρ_m and ρ_Λ behave as H^2 . So the DE results pressureless, since ρ_Λ scales as like as matter energy density ρ_m with the scale factor a as a^{-3} . Also, taking apparent horizon as the IR cut-off may result the constant parameter of state w , which is in contradiction with recent observations implying variable w [16]. In our consideration for non-flat universe, because of the small value of Ω_k we can consider our model as a system which departs slightly from flat space. Consequently we respect the results of flat universe so that we treat apparent horizon only as an arbitrary distance and not as the system's IR cut-off.

On the other hand taking the event horizon, R_h , where

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2} \quad (3)$$

to be the IR cut-off, gives the results compatible with observations for flat universe.

It is fair to claim that simplicity and reasonability of HDE provides more reliable frame to investigate the problem of DE rather than other models proposed in the literature[2, 3, 4]. For instance the coincidence or "why now" problem is easily solved in some models of HDE based on this fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [17]. In fact a suitable evolution of the Universe is obtained when, in addition to the holographic dark energy, an interaction (decay of dark energy to matter) is assumed. Some experimental data has implied that our universe is not a perfectly flat universe and recent papers have favored the universe with spatial curvature [18, 19]. As a matter of fact, we want to remark that although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [20]. Defining the appropriate distance, for the case of non-flat universe has another story. Some aspects of the problem has been discussed in [20, 21]. In this case, the event horizon can not be considered as the system's IR cut-off, because for instance, when the dark energy is dominated and $c = 1$, where c is a positive constant, $\Omega_\Lambda = 1 + \Omega_k$, we find $\dot{R}_h < 0$, while we know that in this situation we must be in de Sitter space with constant EoS. To solve this problem, another distance is considered- radial size of the event horizon measured on the sphere of the horizon, denoted by L - and the evolution of holographic model of dark energy in non-flat universe is investigated.

In present paper, using the holographic model of dark energy in non-flat universe, we obtain equation of state for interacting holographic dark energy density in a universe enveloped by L as the system's IR cut-off.

2 Intracting holographic dark energy density

In this section we obtain the equation of state for the holographic energy density when there is an interaction between holographic energy density ρ_Λ and a Cold Dark Matter(CDM) with $w_m = 0$. The continuity equations for dark energy and CDM are

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q, \quad (4)$$

$$\dot{\rho}_m + 3H\rho_m = Q. \quad (5)$$

The interaction is given by the quantity $Q = \Gamma\rho_\Lambda$. This is a decaying of the holographic energy component into CDM with the decay rate Γ . Taking a ratio of two energy densities as $r = \rho_m/\rho_\Lambda$, the above equations lead to

$$\dot{r} = 3Hr\left[w_\Lambda + \frac{1+r}{r}\frac{\Gamma}{3H}\right] \quad (6)$$

Following Ref.[22], if we define

$$w_\Lambda^{\text{eff}} = w_\Lambda + \frac{\Gamma}{3H}, \quad w_m^{\text{eff}} = -\frac{1}{r}\frac{\Gamma}{3H}. \quad (7)$$

Then, the continuity equations can be written in their standard form

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{\text{eff}})\rho_\Lambda = 0, \quad (8)$$

$$\dot{\rho}_m + 3H(1 + w_m^{\text{eff}})\rho_m = 0 \quad (9)$$

We consider the non-flat Friedmann-Robertson-Walker universe with line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2d\Omega^2\right). \quad (10)$$

where k denotes the curvature of space $k=0,1,-1$ for flat, closed and open universe respectively. A closed universe with a small positive curvature ($\Omega_k \sim 0.01$) is compatible with observations [18, 19]. We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2 + \frac{kc^2}{a^2} = \frac{1}{3M_p^2}[\rho_\Lambda + \rho_m]. \quad (11)$$

Define as usual

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_p^2 H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3M_p^2 H^2}, \quad \Omega_k = \frac{kc^2}{a^2 H^2} \quad (12)$$

Now we can rewrite the first Friedmann equation as

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \quad (13)$$

Using Eqs.(12,13) we obtain following relation for ratio of energy densities r as

$$r = \frac{1 + \Omega_k - \Omega_\Lambda}{\Omega_\Lambda} \quad (14)$$

In non-flat universe, our choice for holographic dark energy density is

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}. \quad (15)$$

As it was mentioned, c is a positive constant in holographic model of dark energy($c \geq 1$) and the coefficient 3 is for convenient. L is defined as the following form:

$$L = ar(t), \quad (16)$$

here, a , is scale factor and $r(t)$ can be obtained from the following equation

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_t^\infty \frac{dt}{a} = \frac{R_h}{a}, \quad (17)$$

where R_h is event horizon. Therefore while R_h is the radial size of the event horizon measured in the r direction, L is the radius of the event horizon measured on the sphere of the horizon.¹ For closed universe we have (same calculation is valid for open universe by transformation)

$$r(t) = \frac{1}{\sqrt{k}} \sin y. \quad (20)$$

where $y \equiv \sqrt{k} R_h / a$. Using definitions $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2 H^2$, we get

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}} \quad (21)$$

Now using Eqs.(16, 17, 20, 21), we obtain²

$$\dot{L} = HL + a\dot{r}(t) = \frac{c}{\sqrt{\Omega_\Lambda}} - \cos y, \quad (23)$$

¹As I have discussed in introduction, in non-flat case the event horizon can not be considered as the system's IR cut-off, because if we use R_h as IR cut-off, the holographic dark energy density is given by

$$\rho_\Lambda = 3c^2 M_p^2 R_h^{-2}. \quad (18)$$

When there is only dark energy and the curvature, $\Omega_\Lambda = 1 + \Omega_k$, and $c = 1$, we find [20]

$$\dot{R}_h = \frac{1}{\sqrt{\Omega_\Lambda}} - 1 = \frac{1}{\sqrt{1 + \Omega_k}} - 1 < 0, \quad (19)$$

while we know that in this situation we must be in de Sitter space with constant EoS.

²Now we see that the above problem is solved when R_h is replaced with L . According to eqs.(12, 15), the ratio of the energy density between curvature and holographic dark energy is

$$\frac{\Omega_k}{\Omega_\Lambda} = \frac{\sin^2 y}{c^2} \quad (22)$$

when there is only dark energy and the curvature, $\Omega_\Lambda = 1 + \Omega_k$, and $c = 1$, we find $\Omega_\Lambda = \frac{1}{\cos^2 y}$, in this case according to eq.(23) $\dot{L} = 0$, therefore, as one expected in this de Sitter space case, the dark energy remains a constant.

By considering the definition of holographic energy density ρ_Λ , and using Eqs.(21, 23) one can find:

$$\dot{\rho}_\Lambda = -2H(1 - \frac{\sqrt{\Omega_\Lambda}}{c} \cos y)\rho_\Lambda \quad (24)$$

Substitute this relation into Eq.(4) and using definition $Q = \Gamma\rho_\Lambda$, we obtain

$$w_\Lambda = -(\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos y + \frac{\Gamma}{3H}). \quad (25)$$

Here as in Ref.[23], we choose the following relation for decay rate

$$\Gamma = 3b^2(1+r)H \quad (26)$$

with the coupling constant b^2 . Using Eq.(22), the above decay rate take following form

$$\Gamma = 3b^2H \frac{(1 + \Omega_k)}{\Omega_\Lambda} \quad (27)$$

Substitute this relation into Eq.(25), one finds the holographic energy equation of state

$$w_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos y - \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda}. \quad (28)$$

If we take $c = 1$, then w_Λ is bounded from below for a fixed b^2 by

$$w_\Lambda = -\frac{1}{3}(1 + 2\sqrt{\Omega_\Lambda} + \frac{3b^2(1 + \Omega_k)}{\Omega_\Lambda}). \quad (29)$$

According to relation $y \equiv \sqrt{k}R_h/a$, $\cos y = 1$ when $k = 0$, in this case $\Omega_k = 0$ also, therefore in flat universe, the holographic energy equation of state take following form

$$w_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} - \frac{b^2}{\Omega_\Lambda}. \quad (30)$$

which is exactly the result of [22]. From Eqs.(7, 27, 28), we have the effective equation of state as

$$w_\Lambda^{eff} = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos y. \quad (31)$$

If we take $c = 1$, and taking $\Omega_\Lambda = 0.73$ for the present time, the lower bound of w_Λ^{eff} is -0.9 . Therefore it is impossible to have w_Λ^{eff} crossing -1 . This implies that one can not generate phantom-like equation of state from an interacting holographic dark energy model in non-flat universe.

3 Conclusions

In order to solve cosmological problems and because the lack of our knowledge, for instance to determine what could be the best candidate for DE to explain the accelerated expansion of universe, the cosmologists try to approach to best results as precise as they can by considering all the possibilities they have. It is of interest to remark that in the literature, the different scenarios of DE has never been studied via considering special

similar horizon, as in [24], in the standard cosmology framworke, the apparent horizon, $1/H$, determines our universe while in [25], in the Brans-Dicke cosmology framworke, the universe is enclosed by event horizon, R_h . As we discussed in introduction, for flat universe the convenient horizon looks to be R_h while in non-flat universe we define L because of the problems that arise if we consider R_h or R_p (these problems arise if we consider them as the system's IR cut-off). In present paper, we studied L , as the horizon measured from the sphere of the horizon as system's IR cut-off. Then, by considering an interaction between holographic energy density and CDM, we have obtained the equation of state for the interacting holographic energy density in the non-flat universe.

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